

II.4-LAG/K LAG AND K ROUTING

Introduction

Lag and K routing provides a computerized solution to a procedure which was initially developed as a graphical routing technique (see Linsley, Kohler and Paulhus, 1975, Section 9.9). Operationally, Lag and K has been and continues to be a practical and widely-used method of storage routing between flow-points. It is also a very flexible method of routing since both the Lag and K elements can be either constant or variable. Examination of historical flood hydrographs of varying magnitude provides a basis for establishing Lag and K relationships within a reach. The first process in a normal operation is to lag or delay in hours an inflow hydrograph in order to create what is called a lagged inflow graph. The K part of the operation is then used to attenuate the lagged inflow graph in order to create an outflow hydrograph at the downstream flow-point. Though normally used together, Lag and K can also be used separately to account for lag with no attenuation or attenuation with negligible lag.

References to computational periods will have different definitions in the Calibration System and the Operational Forecast System. In the Calibration System, a computational period is usually one month. Computational periods in the operational version usually extend from a given period on a given day to a given period on a subsequent day. Periods in this case may transcend the end of a month and extend into the following month.

An alternative K computations method is available. The input is the same for both methods except that the carryover value of storage at the beginning of the routing interval is replaced by the inflow at the previous time interval. Comparisons between the two methods have shown that alternative method is more sensitive to the length of the routing interval.

The Lag Algorithm

The lag algorithm generates a lagged inflow graph by applying either a constant or varying lag in hours to instantaneous inflow values (array QA). After the constant or variable lag is applied, pairs of discharge and time (relative to the start of the execution) are stored in array QT if the time is less than or equal to save carryover time and in array C, if the time is greater than the save carryover time. The pairs of values in the QT array are interpolated to produce a lagged inflow time series with time step equal to the time step of the inflow time series. This lagged inflow time series is stored in array QB. The discharge and time pairs stored in the C array are saved as carryover from computational period to computational period. These values are placed in the beginning of the QT array at the start of the next execution.

The Lag/Inflow Relationship

Constant lag implies that all possible flows are always to be lagged with the same fixed lag time in hours. Only one value of lag needs to be stored in the system when lag is constant. However, variable lag implies that lag in hours has been determined from historical records to be a function of inflow rate. Therefore, a relationship between lag and flow must be stored in the system. The lag versus inflow relationship, known as a variable lag curve, is defined at points and placed in tabular form as shown in Figure 1. Significant changes in the slope of the curve should always be represented with a point in the table. There is no limit within reason to the number of points which can be used. However, the lag versus inflow curve represents an average relationship. It is not exact and from four to ten points should be adequate to describe even the most complex situation. Linear interpolation is used to determine lag for flows which fall between points. Using the relationship provided in Figure 1, lag for an inflow of 180 CMS would be determined as follows:

$$L = 13 + [(11-13) * 180/(200-100)] \quad (1)$$

$$L = 11.4 \text{ hours}$$

The first point in the table should always define a lag time appropriate for zero discharge. The last inflow value in the table implies that greater inflows will have the same lag value. Again using Figure 1 as an example, all inflows greater than 300 CMS will be lagged by 10 hours.

Multiple Intercepts

A problem called the multiple intercept problem may occur with lag versus inflow relationships that are highly variable. Multiple intercepts appear on a lagged inflow graph when the graph doubles back upon itself. Doubling back seemingly causes two or more discharges to occur at the same time or ordinate. Since the lagged inflow graph is a hydrologic tool and a means toward an end which does not really exist in nature (one could never observe a lagged inflow graph at any point along a river), the question of whether or not it is possible to have two or more discharges at the same time is irrelevant. However, it is important to understand how this situation is handled within the lag operation when it does occur.

Figure 2 provides an example of multiple intercepts occurring on both the rising and falling limbs of the lagged inflow graph. Points A through G represent variably lagged instantaneous flows along the rising limb. It can be seen that the 48-hour ordinate is intercepted three times. The method used to handle multiple intercepts tries to maintain the correct volume by adding up the area under the lagged inflow curve at each time ordinate. In this example, at hour 48, the discharge from O to X and the discharge from Y to Z are added to obtain the final lagged inflow graph at hour 48 located at R. The final lagged inflow graph would be represented by the original lagged inflow graph shown in Figure 2 except between points S and T and between U and V where the multiple intercept technique would produce the long and short dashed lines.

Volume errors are possible when multiple intercepts are handled in this manner. Usually they will be minor, as in the example, but significant errors are possible. Peak attenuation is also possible, but only when doubling back on the rising limb occurs directly beneath the peak ordinate. Although this is unlikely, it would severely affect peak simulation if it did occur. As a general comment, alternative routing techniques such as dynamic routing should be considered for flow-points where large errors are caused by the multiple intercept problem.

The Attenuation - K Algorithm

The K part of the Lag and K storage routing procedure is used to attenuate streamflow between upstream and downstream points and produce an outflow hydrograph. K can be thought of as the ratio of storage to discharge. As such, it has the dimension of time and it can be constant or variable. Figure 3 provides an example of an inflow hydrograph which has been lagged and then attenuated by K.

The K/Outflow Relationship

Instead of being related to inflow as lag is, K is related to outflow at the downstream point. K versus outflow relationships at different discharges are determined through investigation of historical flow records. If K is constant throughout the entire range of outflow values, only one value of K is needed for all flows. However, if K varies with outflow, it is necessary to describe a K versus outflow relationship in tabular form. An example is provided in Figure 4. Values in the variable K table are taken from points along the K versus outflow curve. Significant changes in the slope of the curve should always be defined with a point that is represented in the table. Straight line interpolation is used to determine values of K in the same manner as it is done for variable lag. Using the relationship in Figure 4 as an example, a K of 12.8 hours would be assigned to an outflow of 220 CMS.

Storage Routing Equation

Storage routing with K attenuation is based on the continuity equation expressed as:

$$I - O = \frac{ds}{dt} \quad (2)$$

or as

$$\bar{I} - \bar{O} = \frac{\Delta S}{\Delta t} \quad (3)$$

where I is the inflow rate
O is outflow rate
S is storage

t is time with Δt representing the routing interval

Using subscripts 1 and 2 to represent the beginning and end of routing interval Δt , the equation becomes:

$$\frac{I_1 + I_2}{2} \Delta t - \frac{O_1 + O_2}{2} \Delta t = S_2 - S_1 \quad (4)$$

In a typical routing situation, both inflows and the initial outflow and storage values are known. The unknowns in the equation are O_2 and S_2 . By collecting these unknowns, Equation 4 becomes:

$$I_1 + I_2 + \frac{2S_1}{\Delta t} - O_1 = \frac{2S_2}{\Delta t} + O_2 \quad (5)$$

At each time step, the value of the right-hand side of Equation 5 can be determined. If a relationship between $2S_2/\Delta t + O_2$ and O_2 can be found, the routing problem is solved. This relationship can be expressed as a table of O_2 and $2S_2/\Delta t + O_2$ values. At each time step, the value of the right-hand side of Equation 5 can be found and the corresponding O_2 value computed by linear interpolation within the O_2 versus $2S_2/\Delta t + O_2$ table.

Subroutine PINA7 uses the storage equation:

$$S = KO \quad (6)$$

or

$$S_2 - S_1 = \bar{K}(O_2 - O_1) \quad (7)$$

where \bar{K} is the attenuation for the average outflow $(O_1 + O_2)/2$, to develop an O_2 versus $2S_2/\Delta t + O_2$ table.

The number of points in the O_2 versus $2S_2/\Delta t + O_2$ table is computed as:

- a. One point for each point in the variable K table.
- b. Intermediate points are inserted if $\frac{\Delta Q + C1 * \Delta K}{C2} \geq 0.5$

where ΔQ is the change in Q in the variable K table
 ΔK is the change in K in the variable K table

C1 and C2 are empirical constants used to scale ΔQ and ΔK to represent the number of intermediate points needed.

- c. If the smallest Q value in the variable K table is not zero, an entry is made in the O_2 versus $2S_2/\Delta t + O_2$ table for $Q = 0$. (K equals the first K value in the variable K table.)
- d. An entry is made in the O_2 versus $2S_2/\Delta t + O_2$ table for $Q = 10^6$. (K equals the last K value in the variable K table.)

Subroutine KA7 does the attenuation (K) computations each time step. The value of the rhs of Equation 5 is computed and a value for O_2 is found by linear interpolation in the O_2 versus $2S_2/\Delta t + O_2$ table. At this point, several checks are made to determine if the flow conditions are such that the routing method is valid. When a value of O_2 is found which has an associated K that is less than one-half of the routing interval, incorrect results can occur. This contingency is handled in KA7 with the following rules:

- a. If neither K_1 (associated with O_1), nor K_2 (associated with O_2) are less than $\Delta t/2$, the computations proceed as usual.
- b. If either K_1 or K_2 (but not both) is less than $\Delta t/2$, the routing interval is quartered and computations proceed with $\Delta t/4$ for four time periods (the intermediate inflow is found by linear interpolation and only the O_2 produced in the fourth interval is stored as outflow).
- c. If both K_1 and K_2 are less than $\Delta t/2$, O_2 is set equal to the minimum of inflow and the left-hand side (lhs) of Equation 5.

For case b above, within the $\Delta t/4$ computations another table of O_2 versus $2S_2/\Delta t/4 + O_2$ is required. This table will be produced by subroutine PINA7 if the need for it should ever arise. If, in the $\Delta t/4$ computations, either K_1 or K_2 is less than $\Delta t/8$, the outflow is set equal to the minimum of the inflow and the lhs of Equation 5.

The K algorithm can also be used with period average flows instead of instantaneous as derived above. In this case Equation 4 is replaced by:

$$\bar{I} - \frac{O_1 + O_2}{2} \Delta t = S_2 - S_1 \quad (8)$$

and Equation 5 becomes:

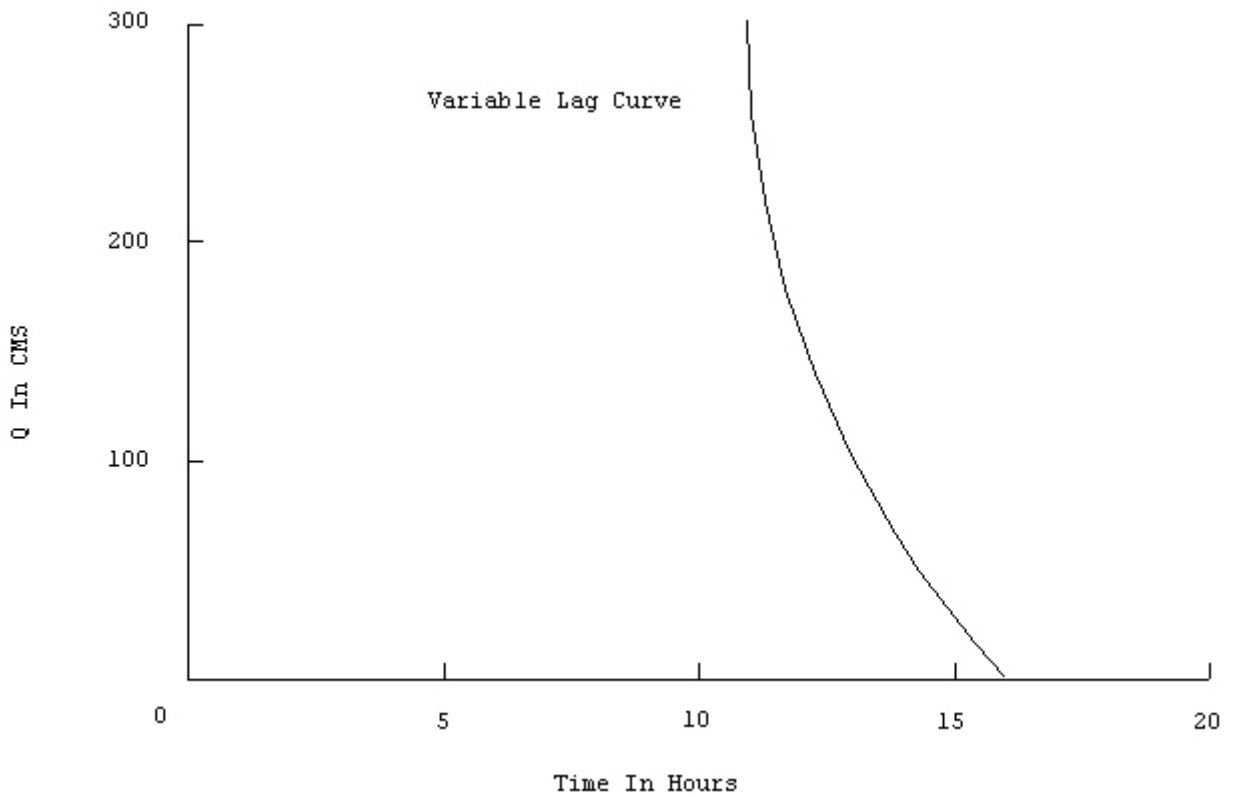
$$\frac{2\bar{I}}{\Delta t} + \frac{2S_1}{\Delta t} - O_1 = \frac{2S_2}{\Delta t} + O_2 \quad (9)$$

When period averaged inflows are used Equation 9 cannot be solved if K is less than one half of the routing interval. If any values of K are read in as less than one half the routing interval, they are reset to one half of the routing interval and a warning is printed.

Reference

Linsley, R.K., Kohler, M.A. and Paulhus, J.L.H., 1975: Hydrology for Engineers, 2nd ed. McGraw Hill Book Company, New York.

Figure 1. Variable Lag versus Inflow Relationship



Variable Lag Table:

Point	Q (CMS)	Lag (HRS)
1	0	16
2	50	14
3	100	13
4	200	11
5	300	10

Figure 2. Lagged Inflow Graph with Multiple Intercepts

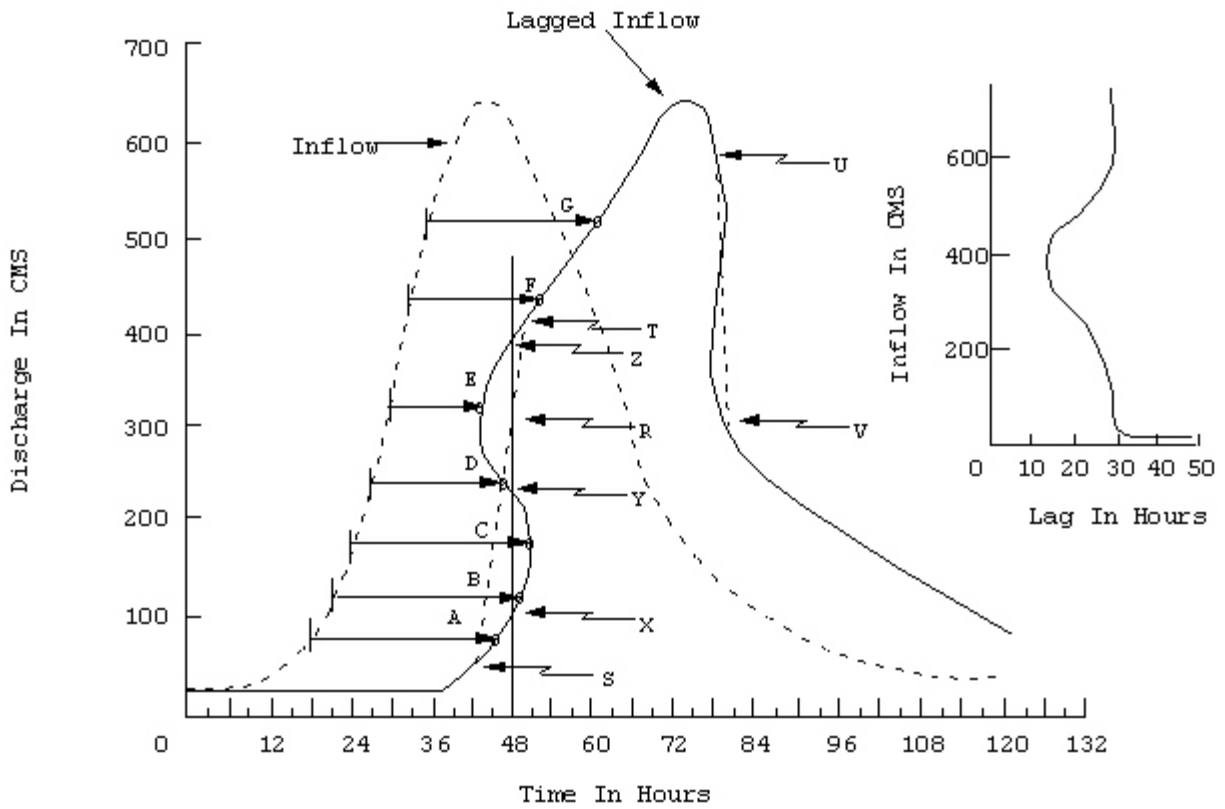


Figure 3. Inflow, Lagged Inflow and Outflow Hydrographs

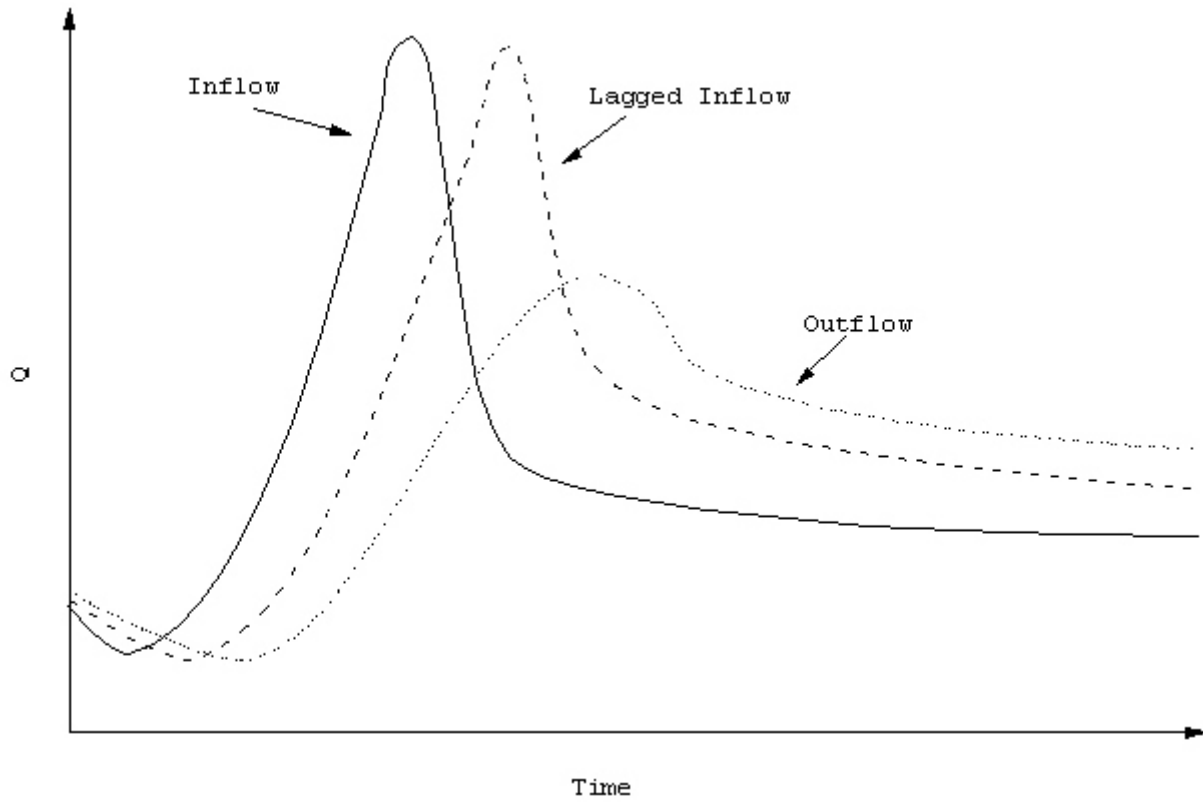
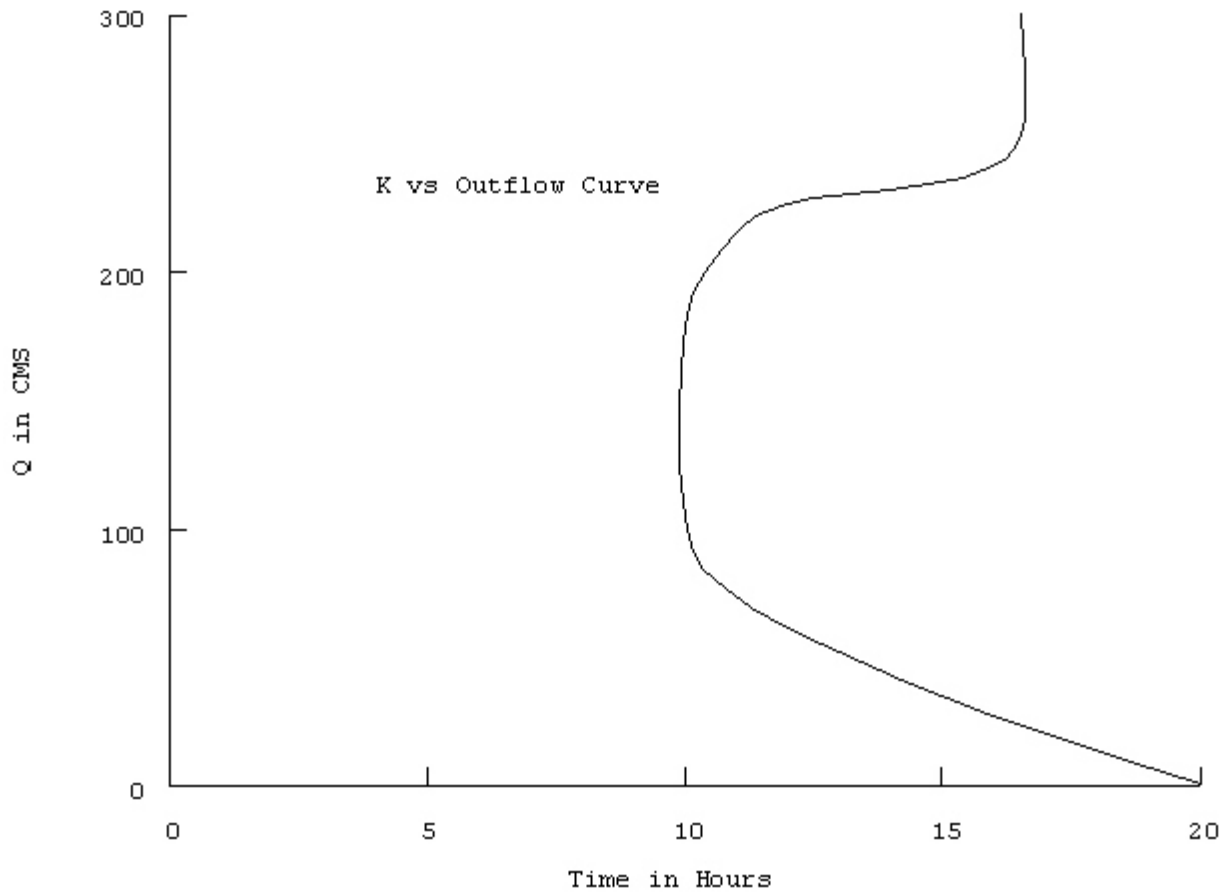


Figure 4. Variable K versus Outflow Relationship



Variable K Table:

Point	Q (CMS)	K (HRS)
1	0	20
2	25	16
3	75	11
4	150	10
5	250	14